Structure of Cosmological CP Violation via Neutrino Seesaw

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The cosmological matter-antimatter asymmetry can originate from CP-violating interactions of seesaw Majorana neutrinos via leptogenesis in the thermal phase of the early universe. Having the cosmological CP phase for leptogenesis requires at least two right-handed Majorana neutrinos. Using only the low energy neutrino observables, we quantitatively reconstruct a minimal neutrino seesaw. We establish a general criterion for minimal seesaw schemes in which the cosmological CP-phase is *completely* reconstructed from the low energy CP-phases measured by neutrino oscillation and neutrinoless double-beta decay experiments. We reveal and analyze two distinct classes of such minimal schemes that are shown to be highly predictive. Extension of our reconstruction formalism to a three-heavy-neutrino seesaw is discussed.

1. Introduction

From Big Bang Nucleosynthesis (BBN) the observed abundance of deuterium determines the baryon asymmetry in the universe to be $\eta_B^{\rm BBN} = (n_B - n_{\overline{B}})/n_\gamma = (6.1^{+0.7}_{-0.5}) \times 10^{-10}$, where n_γ is the current photon number density [1]. The precision of this asymmetry has been improved through measurements of the power spectrum of the cosmic microwave background radiation (CMB). From WMAP (Wilkinson Microwave Anisotropy Probe) data [2] the baryon density $(\Omega_B h^2) = 0.0224 \pm 0.0009$ is inferred, which corresponds to

$$\eta_B^{\rm CMB} = \left(6.5^{+0.4}_{-0.3}\right) \times 10^{-10}\,. \tag{1}$$

The standard model (SM) of particle physics fails to explain the observed baryon asymmetry because the Higgs mass bound from searches at the CERN Large Electron Positron (LEP) collider precludes a sufficiently strong first-order electroweak phase transition [3]. Moreover, only a limited parameter range in the minimal supersymmetric extension of the SM (MSSM) is still viable for baryogenesis.

Alternatively, leptogenesis [4] provides a very attractive mechanism for realistic baryogenesis. This possibility is especially well motivated by the discovery of low energy neutrino oscillations in the past five years (cf. Refs. [5, 6] for recent reviews). The atmospheric neutrino data [7] indicate $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with nearly maximal mixing $\theta_a \simeq 45°$ at the scale of mass-squared-difference $\Delta_a \simeq 2 \times 10^{-3}\,{\rm eV}^2$, while the solar and KamLAND data [8] single out $\nu_e \leftrightarrow \nu_\mu$ oscillations with large mixing angle (LMA) solution of $\theta_s \simeq 32°$ and the scale of mass-squared-difference $\Delta_s \simeq 7 \times 10^{-5}\,{\rm eV}^2$. The CHOOZ and Palo Verde long baseline reactor experiments (in combination with the mass range of the atmospheric oscillation data) constrain the $\nu_e \to \nu_\tau$ transition to be fairly small, $\theta_x \lesssim 13°$ ($\sin\theta_x \lesssim 0.22$) at 95% C.L. [9], which bounds Dirac CP-violation. The seesaw mechanism [10] provides a most natural explanation for the tiny masses of active neutrinos, and the predicted Majorana nature induces two independent Majorana phases at low energy, relevant to neutrinoless double-beta ($0\nu\beta\beta$) decay experiments [11]. In addition, the seesaw mechanism further allows the natural realization of successful leptogenesis, with the light neutrino mass range compatible with the current low energy oscillation data [12].

A crucial ingredient for leptogenesis is the generation of lepton asymmetry via CP-violations in the neutrino seesaw mechanism [4]. Given the excitement from the various current and upcoming neutrino experiments (especially Superbeams and next generation $0\nu\beta\beta$ decay experiments), an intriguing possibility is that *the baryon asymmetry of our universe may be connected to the low energy neutrino CP-violation phenomenon and thus highly testable.* We note that a seesaw sector with k right-handed neutrinos will contain $3(k-1)$ independent CP-phases ($k = 1, 2, 3$). The required CP-violation for leptogenesis can naturally arise from the CP-phases in the seesaw sector, provided $k \geqslant 2$. The Minimal Neutrino Seesaw Models (MNSMs) for leptogenesis contain two heavy Majorana neutrinos (N_1, N_2). Although a link between high energy leptogenesis and low energy neutrino CP-violations is not guaranteed in general [13], in this Letter we will quantitatively derive a *minimal criterion* for the MNSMs under which the high energy leptogenesis CP-asymmetry can be *completely* reconstructed from the low energy neutrino CP-violations. We reveal and analyze two general classes of such MNSMs, and derive the predictions for the allowed ranges of low energy neutrino CP-observables that can generate the observed baryon asymmetry. Finally the extension of our reconstruction formalism to analyzing the leptogenesis in the three neutrino (N_1, N_2, N_3) seesaw is discussed.

2. Leptogenesis CP Asymmetry in the Minimal Neutrino Seesaw

A lepton asymmetry can be dynamically generated

in an expanding universe, provided that all three of Sakharov's conditions [14] are realized: (i) the heavy Majorana neutrinos N_j decay into a lepton-Higgs pair ℓH and its CP-conjugate pair $\overline{\ell} H^*$ with different partial widths, thereby violating lepton number; (ii) CP-violation arises from phases in the seesaw sector; (iii) cosmological expansion causes departure from the thermal equilibrium. As the universe cools down, the N_j's drop out of equilibrium and their decays generate a CP asymmetry,

$$\epsilon_j = \frac{\Gamma[N_j \to \ell H] - \Gamma[N_j \to \overline{\ell} H^*]}{\Gamma[N_j \to \ell H] + \Gamma[N_j \to \overline{\ell} H^*]}. \qquad (2)$$

This leads to a lepton asymmetry,

$$Y_L = \frac{n_L - n_{\overline{L}}}{s} = \sum_j \frac{\epsilon_j \kappa_j}{g_{*j}}, \qquad (3)$$

where g_{*j} represents the relativistic degrees of freedom which contribute to the entropy s at a temperature of the order of the lightest heavy Majorana neutrino mass M_j; in the SM and MSSM, for instance, $g_{*1} = 106.75$ and $g_{*1} = 228.75$, respectively [15]. [Here and below we use SM (MSSM) to denote the usual SM (MSSM) with an addition of the neutrino seesaw sector.] In (3), κ_j describes the wash-out of the asymmetry ϵ_j due to the various lepton-number-violating processes. The wash-out factor κ_j can be computed from the full Boltzmann equations. Analytically κ_1 can be approximated as [15]

$$\kappa_1 \simeq 0.3 \left[\frac{10^{-3}\,\text{eV}}{\widetilde{m}_1} \right] \left[\ln \frac{\widetilde{m}_1}{10^{-3}\,\text{eV}} \right]^{-3/5}, \qquad (4)$$

for $\widetilde{m}_1 = (10^{-2}\text{--}10^3)\,\text{eV}$, where $\widetilde{m}_1 = (m_D^\dagger m_D)_{11}/M_1$ [cf. (6)-(7) below for definitions of m_D and M_1)]. More accurate empirical formulas for κ_1 fitted to the exact solution were also derived [16]. The nonperturbative sphaleron interactions violate $B+L$ but preserve $B-L$ [17], so that the lepton asymmetry Y_L is partially transmitted to a baryon asymmetry Y_B [18]

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} = \frac{\xi}{\xi-1} Y_L = \frac{\xi}{(\xi-1)} \sum_j \frac{\epsilon_j \kappa_j}{g_{*j}}, \quad (5)$$

where $Y_B \simeq \eta_B/7.04$. The parameter ξ depends on the other processes in equilibrium; it is expressed as $\xi = (8N_F + 4N_H)/(22N_F + 13N_H)$ with N_F the number of fermion generations and N_H the number of Higgs doublets. Thus, given the particle contents of the SM (MSSM), we have $\xi = 28/79\,(8/23)$.

The minimal seesaw Lagrangian can be written as

$$\mathcal{L}_{ss} = -\overline{\ell_L} M_\ell \ell_R - \overline{\nu_L} m_D \mathcal{N} + \frac{1}{2} \overline{\mathcal{N}^c}^T M_R \mathcal{N} + \text{h.c.}, \quad (6)$$

where the light neutrinos $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)^T$ are in the flavor eigenbasis, while the charged leptons $\ell = (e, \mu, \tau)^T$ and heavy Majorana neutrinos $\mathcal{N} = (N_1, N_2)^T$ are

in their mass eigenbasis so the mass matrices $M_\ell = \text{diag}(m_e, m_\mu, m_\tau)$ and $M_R = \text{diag}(M_1, M_2)$ are diagonal and real. The 3×2 Dirac mass matrix m_D contains all 3 independent CP-phases in the seesaw sector, and can be generally written as

$$m_D = \begin{pmatrix} a & a' \\ b & b' \\ c & c' \end{pmatrix} = \begin{pmatrix} \zeta_1 \overline{a} & \zeta_2 \overline{a}' \\ \zeta_1 \overline{b} & \zeta_2 \overline{b}' \\ \zeta_1 \overline{c} & \zeta_2 \overline{c}' \end{pmatrix}, \qquad (7)$$

where $\zeta_1 \equiv \sqrt{m_0 M_1}$ and $\zeta_2 \equiv \sqrt{m_0 M_2}$ with $m_0 \equiv \sqrt{\Delta_a}$. In (7) we have defined, for convenience, the 6 dimensionless parameters, $(\overline{a}, \overline{b}, \overline{c}) \equiv (a, b, c)/\sqrt{m_0 M_1}$ and $(\overline{a}', \overline{b}', \overline{c}') \equiv (a', b', c')/\sqrt{m_0 M_2}$, which contain all 3 independent CP-phases. Integrating out the heavy neutrinos from the seesaw Lagrangian (6) we arrive at the symmetric 3×3 Majorana mass matrix for active neutrinos

$$
\begin{aligned}
\mathfrak{M}_\nu &\simeq m_D M_R^{-1} m_D^T \\
&= m_0 \begin{pmatrix} \overline{a}^2 + \overline{a}'^2 & \overline{a}\overline{b} + \overline{a}'\overline{b}' & \overline{a}\overline{c} + \overline{a}'\overline{c}' \\ & \overline{b}^2 + \overline{b}'^2 & \overline{b}\overline{c} + \overline{b}'\overline{c}' \\ & & \overline{c}^2 + \overline{c}'^2 \end{pmatrix}.
\end{aligned}
\qquad (8)
$$

Note that $\mathfrak{M}_\nu$ can be completely expressed in terms of 6 dimensionless parameters $(\overline{a}, \overline{b}, \overline{c})$ and $(\overline{a}', \overline{b}', \overline{c}')$, up to an overall mass scale parameter $m_0 = \sqrt{\Delta_a}$.

We can readily verify that $\det(\mathfrak{M}_\nu) = 0$, so one of the three neutrino mass eigenvalues (m_1, m_2, m_3) must be zero. Therefore the minimal seesaw predicts a *hierarchical neutrino mass spectrum* and excludes a nearly degenerate mass spectrum. The hierarchical mass spectra include the Normal Hierarchy (NH) and Inverted Hierarchy (IH),

$$
\begin{aligned}
\text{NH:} & \quad 0 = m_1 < m_2 \ll m_3, \\
\text{IH:} & \quad m_1 \gtrsim m_2 \gg m_3 = 0,
\end{aligned}
\qquad (9)
$$

where for NH we have $m_2 = m_0\sqrt{r}$, $m_3 = m_0\sqrt{1+r}$; and for IH we have $m_1 = m_0\sqrt{1+r}$, $m_2 = m_0$. Here r is defined as the ratio $r \equiv \Delta_s/\Delta_a$, which is constrained by the oscillation data to the range $1.9 \times 10^{-2} \lesssim r \lesssim 7.4 \times 10^{-2}$ (95% C.L.), with a central value $r \simeq 0.036$.

Now considering thermal leptogenesis with $M_1 \ll M_2$ for simplicity, only the decays of the lightest right-handed neutrino N_1 are relevant for generating the final baryon asymmetry Y_B in (5) because the asymmetry caused by N_2 decays is washed out by the lepton-number-violating processes involving N_1's that are more abundant at the high temperature $T \sim M_2 \gg M_1$. But when temperature $T \sim M_1 \ll M_2$ is reached, the heavier N_2's have already decayed and the asymmetry from N_1 decays is preserved. Thus, under this mechanism, only the term $\epsilon_1 \kappa_1/g_{*1}$ in (5) will contribute. We explicitly derive the

CP-asymmetry parameter ϵ_1 to be

$$\epsilon_1 = \frac{m_0 M_2}{8\pi} \left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 F\left(\frac{M_2}{M_1}\right) \overline{\epsilon}_1 \,,$$

$$\overline{\epsilon}_1 \equiv \frac{\Im\left[\left((m_D^\dagger m_D)_{12}\right)^2\right]}{(m_0 M_2)(m_D^\dagger m_D)_{11}} = \frac{\Im\left[(\overline{a}^*\overline{a}' + \overline{b}^*\overline{b}' + \overline{c}^*\overline{c}')^2\right]}{|\overline{a}|^2 + |\overline{b}|^2 + |\overline{c}|^2}\,, \tag{10}$$

where $\sin\beta = 1\,(<1)$ for SM (MSSM), and the function $F(x)$ is given by [12]

$$F(x) = \begin{cases} x\left[1 - (1+x^2)\ln\dfrac{1+x^2}{x^2} + \dfrac{1}{1-x^2}\right], & \text{(SM)}, \\[2ex] x\left[-\ln\dfrac{1+x^2}{x^2} + \dfrac{2}{1-x^2}\right], & \text{(MSSM)}. \end{cases} \tag{11}$$

The CP-asymmetry parameter $\overline{\epsilon}_1$ in (10) is solely expressed as a function of the six dimensionless parameters $(\overline{a}, \overline{a}', \overline{b}, \overline{b}', \overline{c}, \overline{c}')$ introduced for the Dirac mass matrix m_D in Eq. (7). For the case $M_1 \gg M_2$, the relevant CP-asymmetry is ϵ_2 from N_2 decays and can be independently analyzed in the same way as the reconstruction analysis of ϵ_1 below.

3. Structure of Cosmological CP Violation from Low Energy Reconstruction

● *Reconstructing the 3×3 Neutrino Mass Matrix*

Consider a generic 3×3 symmetric Majorana mass matrix $\mathfrak{M}_\nu$ for 3 light flavor-neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$,

$$\mathfrak{M}_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ & m_{\mu\mu} & m_{\mu\tau} \\ & & m_{\tau\tau} \end{pmatrix} \equiv m_0 \begin{pmatrix} A & C & D \\ & B & E \\ & & F \end{pmatrix}. \tag{12}$$

The mass matrix (12) contains nine independent real parameters, which can be equivalently chosen as three mass eigenvalues $(m_1, m_2, m_3) \geqslant 0$, three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, and three CP-violation phases $(\delta; \phi, \phi')$ with δ the usual Dirac phase and (ϕ, ϕ') the Majorana phases (irrelevant to neutrino oscillations). For MNSMs we have $\det(\mathfrak{M}_\nu) = 0$ so that there are only two nonzero mass-eigenvalues [cf. (9)]. The neutrino mixing matrix [19] is $V \equiv UU'$, which diagonalizes $\mathfrak{M}_\nu$ via $V^T \mathfrak{M}_\nu V = \mathfrak{M}_\nu^{\text{diag}}$, and contains six parameters (three rotation angles and three phases). The unitary matrix V can be decomposed into a matrix U (à la Cabibbo-Kobayashi-Maskawa) and a diagonal matrix U' with two independent Majorana phases, i.e.,

$$U = \begin{pmatrix} c_s c_x & -s_s c_x & -s_x e^{i\delta} \\ s_s c_a - c_s s_a s_x e^{-i\delta} & c_s c_a + s_s s_a s_x e^{-i\delta} & -s_a c_x \\ s_s s_a + c_s c_a s_x e^{-i\delta} & c_s s_a - s_s c_a s_x e^{-i\delta} & c_a c_x \end{pmatrix}. \tag{13}$$

where for convenience we have introduced the notation $(\theta_s, \theta_a, \theta_x) \equiv (\theta_{12}, \theta_{23}, \theta_{13})$, with $(s_\alpha, c_\alpha) \equiv (\sin\theta_\alpha, \cos\theta_\alpha)$ for $\alpha = s, a, x$. The matrix U' can be parametrized as, $U' = \text{diag}(e^{-i\phi'/2}, e^{-i\phi/2}, 1)$ for NH schemes and $U' = \text{diag}(1, e^{-i\phi/2}, e^{-i\phi'/2})$ for IH schemes. [Our convention of V is related to that of particle data book via $(\theta_{s,a,x}, \delta, \phi, \phi') \to -(\theta_{s,a,x}, \delta, \phi, \phi')$.] Because of the mass spectrum of the minimal seesaw in Eq. (9), we see that in the above parametrization of U' only a single Majorana phase ϕ is physically relevant. From the mass diagonalization, we can reconstruct the neutrino mass matrix $\mathfrak{M}_\nu$ via the relation

$$\mathfrak{M}_\nu = V^* \mathfrak{M}_\nu^{\text{diag}} V^\dagger \,, \tag{14}$$

which gives

$$\begin{aligned} m_{ee} &= c_x^2\left[c_s^2 m_1' + s_s^2 m_2'\right] + \bar{p}^2 s_x^2 m_3'\,, \\ m_{\mu\mu} &= (s_s c_a - p\, c_s s_a s_x)^2 m_1' + (c_s c_a + p\, s_s s_a s_x)^2 m_2' \\ &\quad + s_a^2 c_x^2 m_3'\,, \\ m_{\tau\tau} &= (s_s s_a + p\, c_s c_a s_x)^2 m_1' + (c_s s_a - p\, s_s c_a s_x)^2 m_2' \\ &\quad + c_a^2 c_x^2 m_3'\,, \\ m_{e\mu} &= c_x\left[s_s c_s c_a(m_1' - m_2') - p\, s_a s_x(c_s^2 m_1' + s_s^2 m_2')\right. \\ &\quad \left. + \bar{p}\, s_a s_x m_3'\right]\,, \\ m_{e\tau} &= c_x\left[s_s c_s s_a(m_1' - m_2') + p\, c_a s_x(c_s^2 m_1' + s_s^2 m_2')\right. \\ &\quad \left. - \bar{p}\, c_a s_x m_3'\right]\,, \\ m_{\mu\tau} &= (s_s s_a + p\, c_s c_a s_x)(s_s c_a - p\, c_s s_a s_x)m_1' \\ &\quad + (c_s s_a - p\, s_s c_a s_x)(c_s c_a + p\, s_s s_a s_x)m_2' - s_a c_a c_x^2 m_3'. \end{aligned} \tag{15}$$

For NH schemes $(m_1', m_2', m_3') \equiv (0, m_2 q, m_3) = m_0(0, \sqrt{r}q, \sqrt{1+r})$, for IH schemes $(m_1', m_2', m_3') \equiv (m_1, m_2 q, 0) = m_0(\sqrt{1+r}, q, 0)$, and the two relevant CP-phase factors are $(p, q) = (\bar{p}^*, q) = (e^{i\delta}, e^{i\phi})$.

Unlike most other bottom-up approaches, our reconstruction formalism solely relies on the low energy neutrino observables without any extra ansatz about the specific forms of the neutrino mass matrix (12), and is thus completely general for a given seesaw sector.

● *Reconstruction Theorem in Minimal Neutrino Seesaw*

From (8) and (12), we derive the reconstruction equation:

$$\begin{pmatrix} \overline{a}^2 + \overline{a}'^2 & \overline{a}\overline{b} + \overline{a}'\overline{b}' & \overline{a}\overline{c} + \overline{a}'\overline{c}' \\ & \overline{b}^2 + \overline{b}'^2 & \overline{b}\overline{c} + \overline{b}'\overline{c}' \\ & & \overline{c}^2 + \overline{c}'^2 \end{pmatrix} = \begin{pmatrix} A & C & D \\ & B & E \\ & & F \end{pmatrix}. \tag{16}$$

This equation contains 6 (complex) conditions among which 5 are independent and the sixth condition is redundant because of $\det(\mathfrak{M}_\nu) = 0$ in the minimal seesaw. Accordingly, the property $\det(\mathfrak{M}_\nu) = 0$ also makes one of the 6 elements (A, B, C, D, E, F) to be dependent. Therefore we can analytically solve (reconstruct) 5 out of

the 6 dimensionless seesaw parameters $(\overline{a},\, \overline{a}',\, \overline{b},\, \overline{b}',\, \overline{c},\, \overline{c}')$ of m_D in terms of the low energy neutrino observables contained in (A, B, C, D, E, F). For $A \neq 0$, we can exactly resolve (16) in terms of $\overline{a}$ or $\overline{a}'$,

$$
\begin{aligned}
\overline{a}' &= \widehat{s}_{a'}\sqrt{A - \overline{a}^2}\,, \quad \text{or,} \quad \overline{a} = \widehat{s}_a \sqrt{A - \overline{a}'^2}\,, \\
\overline{b} &= \frac{1}{A}\left[\overline{a}\,C - \widehat{s}_{b'}\overline{a}'\sqrt{AB - C^2}\right], \\
\overline{b}' &= \frac{1}{A}\left[\overline{a}'\,C + \widehat{s}_{b'}\overline{a}\sqrt{AB - C^2}\right], \\
\overline{c} &= \frac{1}{A}\left[\overline{a}\,D - \widehat{s}_{c'}\overline{a}'\sqrt{AF - D^2}\right], \\
\overline{c}' &= \frac{1}{A}\left[\overline{a}'\,D + \widehat{s}_{c'}\overline{a}\sqrt{AF - D^2}\right], \\
E &= \frac{1}{A}\left[CD + \widehat{s}_{b'}\widehat{s}_{c'}\sqrt{(AB - C^2)(AF - D^2)}\right],
\end{aligned}
\tag{17}
$$

where $(\widehat{s}_a,\, \widehat{s}_{a'},\, \widehat{s}_{b'},\, \widehat{s}_{c'}) = \pm 1$. We see that with a single input of either $\overline{a}$ or $\overline{a}'$ the other 5 dimensionless parameters in m_D are completely determined from the low energy neutrino observables contained in (A, B, C, D, F); the last equation is a consistency condition due to $\det(\mathfrak{M}_\nu) = 0$ that will also fix the sign convention of $\widehat{s}_{b'}\widehat{s}_{c'}$ from the reconstructed 3×3 low energy neutrino mass matrix $\mathfrak{M}_\nu$ in (12) and (15). Noting that the reconstruction equation (16) is invariant under the joint transformations $\overline{a} \leftrightarrow \overline{b}$, $\overline{a}' \leftrightarrow \overline{b}'$, $A \leftrightarrow B$, $D \leftrightarrow E$, we can readily resolve (16) in terms of $\overline{b}$ or $\overline{b}'$, for $B \neq 0$. Similarly, using the invariance of (16) under the exchanges $\overline{a} \leftrightarrow \overline{c}$, $\overline{a}' \leftrightarrow \overline{c}'$, $A \leftrightarrow F$, $C \leftrightarrow E$, we can reexpress the solutions in terms of $\overline{c}$ or $\overline{c}'$ instead, for $F \neq 0$, and so on. The exact solution (17) explicitly shows that from the low energy neutrino observables in $\mathfrak{M}_\nu$ we can reconstruct 5 out of the 6 dimensionless parameters in (7) for m_D. In the minimal seesaw m_D contains a maximal 6 CP-phases among which only 3 combinations are independent, and one independent combination gives the required CP-phase for leptogenesis. Hence, we can formulate a Reconstruction Theorem for leptogenesis:

In the minimal neutrino seesaw models (MNSMs) with the leptons and heavy Majorana neutrinos in their mass-eigenbasis, five out of six dimensionless parameters $(\overline{a},\, \overline{a}',\, \overline{b},\, \overline{b}',\, \overline{c},\, \overline{c}')$ in m_D can be completely reconstructed from the low energy neutrino observables. A complete reconstruction can be realized in a class of MNSMs where one of the six entries in m_D is not independent. The simplest such MNSMs can be classified into Type-I and Type-II, where Type-I has 1-texture-zero and Type-II has 1-equality between two entries of m_D. Consequently, for Type-I and -II, all CP-phases in m_D are fully reconstructable from low energy, so that a unique link is realized between the high energy leptogenesis CP-asymmetry and the low energy neutrino CP-violation observables.

In fact the requirement of 1-texture-zero or 1-equality is the *Minimal Criterion* for a complete reconstruction of the seesaw sector (up to two mass parameters $M_{1,2}$ of N_1 and N_2). We note that in the above reconstruction

the CP-phase distribution in all 6 entries of m_D is also fully fixed, and there is no more freedom for rephasing of lepton-doublets in the Yukawa interaction or Dirac mass term because we have made use of the rephasing of all three lepton-doublets to derive the standard parametrization of the MNSP mixing matrix $V = UU'$ [19] in the leptonic charged currents such that only 3 independent CP-phases (δ, ϕ, ϕ') enter our low energy formulation (where one of the two Majorana phases (ϕ, ϕ') becomes irrelevant in the MNSMs). A unique feature of our reconstruction formalism is that it does not introduce any extra unphysical CP-phase beyond the standard MNSP mixing matrix V [19], so it has great physical transparency and technical simplicity.

Based upon the above reconstruction formulation, we can further define and classify the Most Minimal Neutrino Seesaw Models (MMNSMs) in which m_D conatins 2-texture-zeros or 2-equalities or 1-texture-zero plus 1-equality, so that a *single* CP-phase alone generates all CP-violations at both high and low energies. (Our classification of the MMNSMs include the recent Frampton-Glashow-Yanagida leptogenesis scheme [20] as one subclass.) A complete exploration of the MMNSMs will be given elsewhere [21].

● *Classifying Minimal Schemes for Leptogenesis*

According to the Reconstruction Theorem above, we explicitly classify our Type-I and Type-II MNSMs. For Type-I class of MNSMs, there are six 1-texture-zero schemes for m_D in total,

$$
m_D^I = \begin{pmatrix} a & 0 \\ b & b' \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ b & 0 \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ b & b' \\ c & 0 \end{pmatrix},
$$
$$
\begin{pmatrix} 0 & a' \\ b & b' \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ 0 & b' \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ b & b' \\ 0 & c' \end{pmatrix},
\tag{18}
$$

which we label as Type-Ia through Type-If. For the Type-II MNSMs, there are 15 schemes depending on how the equality is chosen, which can be horizontal (such as $a = a'$), or vertical (such as $b = c$), or crossing (such as $b = c'$). The Horizontal Equalities (HE) in m_D are *invariant* under rephasing and are thus not affected when we fix the low energy parametrization of MNSP matrix in the leptonic charged currents. The Vertical Equalities (VE) or Crossing Equalities (CE) in m_D arising from an underlying theory could be changed by the rephasing so they are less appealing in comparison with the horizontal equalities. In the following we focus on the analysis of Type-II schemes with *horizontal equality*. There are only 3 such Type-II-HE schemes,

$$
m_D^{II} = \begin{pmatrix} a & \pm a \\ b & b' \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ b & \pm b \\ c & c' \end{pmatrix},\quad
\begin{pmatrix} a & a' \\ b & b' \\ c & \pm c \end{pmatrix},\quad
\tag{19}
$$

where in each scheme we can have a variation scheme by simply flipping the sign of the equality.

4. Reconstruction Analysis for Leptogenesis

We start by proving a general theorem about the structure of the leptogenesis in the two-heavy-neutrino seesaw schemes. The theorem will establish the connection between a nonzero leptogenesis (due to $\epsilon_1 \neq 0$) and a nonzero solar mass-squared-difference (due to $\Delta_s \neq 0$). The *Part-I* of this theorem states that *for any two-heavy-neutrino seesaw with IH, nonzero leptogenesis can be realized only if the low energy observable Δ_s is not zero (because $\epsilon_1 \to 0$ as $r \equiv \Delta_s/\Delta_a \to 0$)*. To prove this, we derive, for *any* two-heavy-neutrino seesaw under $r \to 0$,

$$
\begin{aligned}
&\left[\overline{a}^*\overline{a}' + \overline{b}^*\overline{b}' + \overline{c}^*\overline{c}'\right]_{\text{IH}} \\
&= \frac{c_x^2}{|A|^2}\left[(\overline{a}^*\overline{a}' - \overline{a}\,\overline{a}'^*) + \widehat{s}_{b'}s_s c_s(|\overline{a}|^2 + |\overline{a}'|^2)(q^{\frac{1}{2}} - q^{-\frac{1}{2}})\right] \\
&= \text{Imaginary}, \quad \Rightarrow \quad \epsilon_1 = 0,
\end{aligned}
\tag{20}
$$

where $|A| \neq 0$ is ensured by the ν-oscillation data.

We next consider the NH scenario. Using our reconstruction solution (17) and its variations, and defining $\Sigma \equiv [\overline{a}^*\overline{a}' + \overline{b}^*\overline{b}' + \overline{c}^*\overline{c}']_{\text{NH}}$, we deduce, for $r \to 0$,

$$
\Sigma = \frac{\overline{c}^*\overline{c}'}{c_a^2 c_x^2} = \frac{\overline{b}^*\overline{b}'}{s_a^2 c_x^2} = \frac{\overline{a}^*\overline{a}'}{s_x^2},
\tag{21}
$$

where the last equality is derived for $s_x \neq 0$. [The combined limit $(r, s_x) \to 0$ will force $(\overline{a}, \overline{a}') \to 0$ and a vanishing solar angle $\theta_s \to 0$, but this is not our concern since we examine $r \to 0$ as an independent limit with all mixing angles confined to their physical ranges.] Eq. (21) shows that for any m_D with 1-texture-zero or 1-horizontal-equaltiy, we have, under the $r \to 0$ limit, $\Sigma = 0$ or $\Sigma = $ Real, which means $\epsilon_1 = 0$. Hence, the *Part-II* of our theorem states: *For NH, any viable 2-heavy-neutrino seesaw scheme with 1-texture-zero or 1-horizontal-equaltiy in m_D can have nonzero leptogenesis only if the low energy observable Δ_s is not zero ($r \neq 0$).*

4.1. Analysis for Type-I Minimal Schemes

We next turn to explicit analysis of the Type-Ia scheme in (18) with $a' = 0$. From (17) the complete reconstruction solution is given by

$$
\begin{aligned}
\overline{a} &= \widehat{s}_a\sqrt{A}, \quad \overline{a}' = 0, \\
\overline{b} &= \widehat{s}_a\frac{C}{\sqrt{A}}, \quad \overline{b}' = \widehat{s}_a\widehat{s}_{b'}\sqrt{\frac{AB - C^2}{A}}, \\
\overline{c} &= \widehat{s}_a\frac{D}{\sqrt{A}}, \quad \overline{c}' = \widehat{s}_a\widehat{s}_{c'}\sqrt{\frac{AF - D^2}{A}}.
\end{aligned}
\tag{22}
$$

We thus derive the CP-asymmetry $\overline{\epsilon}_1$ from (10),

$$
\overline{\epsilon}_1 = \frac{\Im\mathfrak{m}\left[\left(C^*\sqrt{AB - C^2} + \widehat{s}_{b'}\widehat{s}_{c'}D^*\sqrt{AF - D^2}\right)^2\right]}{|A|\left(|A|^2 + |C|^2 + |D|^2\right)},
\tag{23}
$$

which is expressed solely in terms of the low energy neutrino observables [cf. Eqs. (12) and (15)].

● *Type-I Schemes with Inverted Hierarchy*

For the IH scenario, we derive for Type-Ia ($a' = 0$),

$$
\overline{\epsilon}_1 = \frac{s_s^2 c_s^2 \sqrt{1+r}(1 + rc_s^2)^{-1}(r\sin\phi)}{\left[(c_s^2\sqrt{1+r} + s_s^2)^2 + 2s_s^2 c_s^2(\cos\phi - 1)\sqrt{1+r}\right]^{1/2}}.
\tag{24}
$$

Strikingly, $\overline{\epsilon}_1$ depends on Majorana phase ϕ only; it is independent of θ_x and Dirac phase δ.

Type-Id ($a = 0$) is very similar to Type-Ia; $\overline{\epsilon}_1$ is given by (24) with the factor $\sqrt{1+r}(1 + rc_s^2)^{-1}$ replaced by -1. We have also derived the expressions for other Type-I schemes, which take simple analytical forms under the expansion in $\lambda \equiv O(\sqrt{r}, s_x)$. For Type-Ib ($b' = 0$) and Type-Ie ($b = 0$), we find

$$
\begin{aligned}
\overline{\epsilon}_1 &= \pm \frac{rs_s c_s\left[s_s c_s c_a\sin\phi + 2s_a s_x z_1\right]}{c_a\left[1 + 2s_s^2 c_s^2(\cos\phi - 1)\right]^{1/2}} + O(\lambda^4), \\
z_1 &\equiv c_s^2\sin(\delta - \phi) + s_s^2\sin(\delta + \phi),
\end{aligned}
\tag{25}
$$

which are numerically very accurate. In (25) the overall $+(-)$ sign corresponds to Type-Ib (-Ie) scheme.

For Type-Ic ($c' = 0$) and Type-If ($c = 0$), $\overline{\epsilon}_1$ is given by (25) with the replacements, $s_a \leftrightarrow c_a$ and $s_x \to -s_x$. Interestingly the CP-asymmetry $\overline{\epsilon}_1$ is independent of θ_x and the Dirac phase δ (Type-Ia and -Id), or depends on them only at higher orders. This means that in the Type-I schemes with inverted hierarchy, leptogenesis is controlled by a single CP-phase ϕ which also appears in the low energy $0\nu\beta\beta$ decay observable,

$$
|\mathfrak{M}_{ee}| = m_0 c_x^2\left[\left(c_s^2\sqrt{1+r} + s_s^2\right)^2 - \sqrt{1+r}\sin^2 2\theta_s\sin^2\frac{\phi}{2}\right]^{\frac{1}{2}}.
\tag{26}
$$

Hence the Type-I schemes with IH predict an *unique link* between high energy and low energy CP-violations via a *single phase ϕ*. But, given the anticipated future uncertainty in the nuclear matrix elements[11, 22], the small size of the predicted $|\mathfrak{M}_{ee}|$, and the expected uncertainty in the eventual cosmological determination of the sum of neutrino masses for the hierarchical neutrino spectrum[23], it is hard to probe the Majorana phase ϕ [24]. Our study of the low energy test of leptogenesis provides a motivation for greatly improving the $|\mathfrak{M}_{ee}|$ determination; such efforts are currently underway[22].

● *Type-I Schemes with Normal Hierarchy*

For NH scenario, we derive for Type-Ia,

$$\overline{\epsilon}_1 = \frac{s_s^2 s_x^2 c_x^2 \sqrt{r(1+r)}\,\sin(\phi+2\delta)}{[r(1-c_s^2 c_x^2)+s_x^2]\,z_2}$$

$$= \frac{s_s^2 s_x^2 \sin(\phi+2\delta)}{[rs_s^2+s_x^2]\,[s_s^2+\cos(\phi+2\delta)s_x^2/\sqrt{r}]} + O(\lambda^3),$$

$$z_2 \equiv \Big[\big(c_x^2 s_s^2\sqrt{r}+s_x^2\sqrt{1+r}\big)^2 +$$
$$2\sqrt{r(1+r)}s_s^2 s_x^2 c_x^2\,(\cos(\phi+2\delta)-1)\Big]^{\frac{1}{2}},$$

(27)

where $\lambda \equiv O(\sqrt{r},\,s_x)$. Type-Id is again very similar to Type-Ia except that in (27) the denominator of $\overline{\epsilon}_1$ will have its first brackets $[\cdots]$ replaced by $-[s_s^2+c_s^2 s_x^2]$. Unlike the IH case, the $\overline{\epsilon}_1$ is now controlled by a single phase-combination $\phi+2\delta$ from low energy. Interestingly, we find that the $0\nu\beta\beta$ decay observable now becomes

$$|\mathfrak{M}_{ee}| = m_0\big[s_s^2\sqrt{r}+s_x^2\cos(\phi+2\delta)+O(\lambda^3)\big]. \quad (28)$$

which depends on the *same* phase-combination $\phi+2\delta$. Furthermore, the Dirac phase δ can be measured at low energy in long-baseline neutrino oscillation experiments via the leptonic Jarlskog invariant [25],

$$\mathcal{J} = \frac{1}{8}\sin2\theta_s\sin2\theta_a\sin2\theta_x\cos\theta_x\sin\delta. \quad (29)$$

The results of $\overline{\epsilon}_1$ in other Type-I schemes are not so simple for the normal hierarchy. For Type-Ib, we find

$$\overline{\epsilon}_1 = \frac{\sqrt{r}c_s c_a\,[c_s c_a\sin\phi+2s_s s_a s_x\sin(\phi+\delta)]}{s_a^2+\sqrt{r}c_s^2 c_a^2\cos\phi} + O(\lambda^3),$$

(30)

while for Type-Ie, $\overline{\epsilon}_1$ is given by

$$\frac{-s_a^2 c_s c_a\,[c_s c_a\sin\phi+2s_s s_a s_x\sin(\phi+\delta)]}{[c_s^2+s_s^2 s_a^2+2s_s c_s s_a c_a s_x\cos\delta][s_a^2+\sqrt{r}c_s^2 c_a^2\cos\phi]} + O(\lambda^2).$$

(31)

For $c'=0$ (Type-Ic) or $c=0$ (Type-If), $\overline{\epsilon}_1$ is given by (30) or (31) with the replacements $s_a \leftrightarrow c_a$ and $s_x \rightarrow -s_x$.

We may mention that the phase convention below (15) is not unique. If we assign the independent Majorana phase instead to the mass m_3' (NH), or m_1' (IH), the results for $\overline{\epsilon}_1$ would change only by $\phi \rightarrow -\phi$ everywhere.

Finally we analyze the predictions for leptogenesis in the Type-Ia schemes [cf. Fig. 1]. For the IH scenario, we first show the asymmetry η_B versus the low energy Majorana CP-phase ϕ, where we set $x=M_2/M_1=10$. [The result is very insensitive to x for $x\geqslant 3$, because in the SM and MSSM, $xF(x)$ equals $-\frac{3}{2}$ and -3, to 7% (2%) accuracy or better for $x\geqslant 3$ ($x\geqslant 5$).] Fig. 1(a) shows that for the best fit values of ν-oscillation parameters the CMB limit (1) constrains $M_1\gtrsim 10^{13}$ GeV for IH, and for $M_1\geqslant 1.3\times10^{13}$ GeV the phase ϕ is confined to two narrow regions. In Fig. 1(b), we plot the physical mass M_1 vs. the low energy observable $|\mathfrak{M}_{ee}|$ of $0\nu\beta\beta$ decay which are both controlled by the same CP-phase ϕ. Since ϵ_1 is sensitive to (r,m_0), we have scanned the 95% C.L. range of (Δ_s,Δ_a) (and also θ_s) in addition to varying ϕ, and

found that M_1 is always above 4×10^{12} GeV, but has little chance to go above 2×10^{14} GeV. Future improved precision of (Δ_s,Δ_a) will strengthen the bound.

For the NH scenario, we first plot η_B as a function of s_x by varying the CP-phase angle $\phi+2\delta$ [cf. Fig. 1(c)]. Then, in Fig. 1(d) we analyze the correlation between the scale M_1 and the low energy CP-observable J in (29) by varying both phases $(\phi,\delta)\in(0,2\pi)$. We have scanned the 95% C.L. ranges of all ν-oscillation parameters, together with η_B. Distinctly, we find $M_1\geqslant 3\times10^{10}$ GeV, and for $|J|\geqslant 0.01$, M_1 is mainly below 10^{12} GeV.

4.2. Analysis for Type-II Minimal Schemes

Next we turn to the analysis of the Type-II schemes with horizontal equality. We consider the Type-II-HEa Schemes with the equality $a=a'$. From (17) we thus derive a complete reconstruction solution,

$$\overline{a}=\widehat{s}_a\frac{\sqrt{A}}{\omega_2},\qquad \overline{a}'=\widehat{s}_a\frac{\sqrt{A}}{\omega_1},$$

$$\overline{b}=\widehat{s}_a\frac{1}{\omega_2\sqrt{A}}\Big[C-\widehat{s}_{b'}\rho^{-1}\sqrt{AB-C^2}\Big],$$

$$\overline{b}'=\widehat{s}_a\frac{1}{\omega_1\sqrt{A}}\Big[C+\widehat{s}_{b'}\rho\sqrt{AB-C^2}\Big], \quad (32)$$

$$\overline{c}=\widehat{s}_a\frac{1}{\omega_2\sqrt{A}}\Big[D-\widehat{s}_{c'}\rho^{-1}\sqrt{AF-D^2}\Big],$$

$$\overline{c}'=\widehat{s}_a\frac{1}{\omega_1\sqrt{A}}\Big[D+\widehat{s}_{c'}\rho\sqrt{AF-D^2}\Big],$$

where we have introduced the notation $\rho\equiv\sqrt{M_2/M_1}$, $\omega_1=\sqrt{1+\rho^2}$, and $\omega_2=\sqrt{1+\rho^{-2}}$. The CP-asymmetry observable $\overline{\epsilon}_1$ is given by

$$\overline{\epsilon}_1 = \frac{2(\rho+\rho^{-1})}{\omega_1^2|A|}\frac{\widehat{s}_{b'}X+(\rho-\rho^{-1})Y_1}{X_1+\rho^{-2}X_2-\widehat{s}_{b'}2\rho^{-1}Y_1}Y_2, \quad (33)$$

where

$$X \equiv X_1-X_2$$
$$= \big[|A|^2+|C|^2+|D|^2\big]-\big[|AB-C^2|+|AF-D^2|\big],$$

$$Y \equiv Y_1+iY_2$$
$$= C^*\sqrt{AB-C^2}+\widehat{s}_{b'}\widehat{s}_{c'}D^*\sqrt{AF-D^2},$$

(34)

and $\widehat{s}_{b'}\widehat{s}_{c'}=+(-)$ for IH (NH) Schemes.

● *Type-II Schemes with Inverted Hierarchy*

For the IH scenario, we derive the CP-asymmetry of leptogenesis for Type-II-HEa Schemes,

$$\overline{\epsilon}_1 = \frac{2s_s c_s\Big[\widehat{s}_{b'}(c_s^2-s_s^2)+(\rho-\rho^{-1})s_s c_s\cos\frac{\phi}{2}\Big]}{(\rho+\rho^{-1})\Big[1-4s_s^2 c_s^2\sin^2\frac{\phi}{2}\Big]^{1/2}}\Big(r\sin\frac{\phi}{2}\Big),$$
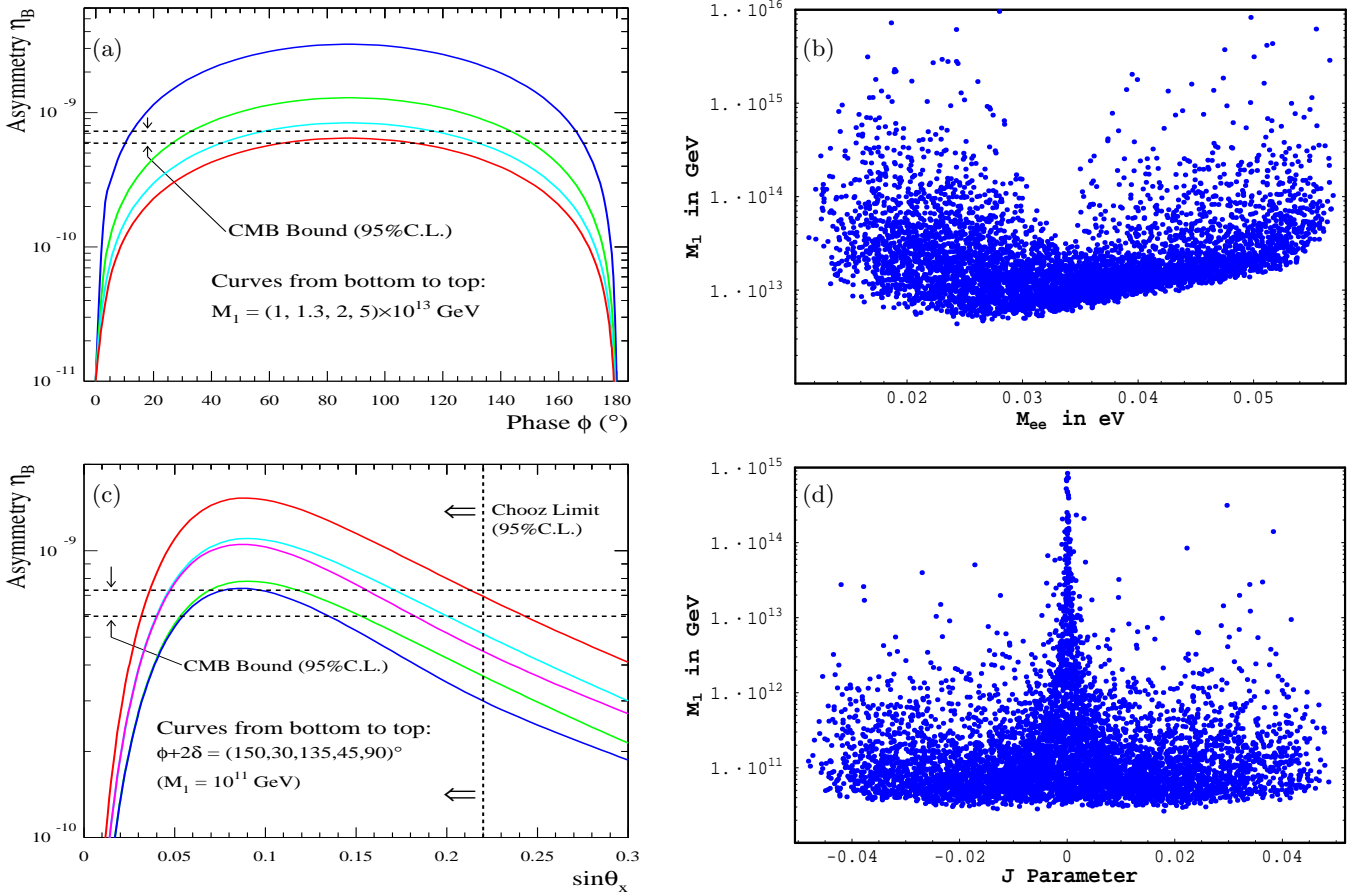
(35)

FIG. 1: Analysis for Type-Ia. (a) Baryon asymmetry η_B vs. the low energy Majorana phase ϕ in the IH scenario, with best-fit values of ν-oscillation observables as inputs. (b) Leptogenesis scale M_1 vs. the $0\nu\beta\beta$-decay observable M_{ee} ($\equiv |\mathfrak{M}_{ee}|$) with IH, where we have varied the phase $\phi \in (0, 2\pi)$, imposed the 95% C.L. CMB limit and scanned the 95% C.L. ranges of (Δ_s, Δ_a, θ_s, s_x), for 1.2×10^4 samples. (Here s_x is relevant for $|\mathfrak{M}_{ee}|$, but varying s_x within $|s_x| \leqslant 0.22$ has only minor effect on $|\mathfrak{M}_{ee}|$ because $c_x^2 > 0.95 \simeq 1$.) (c) Asymmetry η_B vs. $\sin\theta_x$ in the NH scenario, with best-fit values of the ν-oscillation observables as inputs. (d) Mass scale M_1 vs. the CP-violation observable J in the NH scenario, where we have varied (δ, ϕ) $\in (0, 2\pi)$, imposed the CMB limit and scanned the ν-oscillation parameters in their 95% C.L. ranges, for 1.2×10^4 samples.

where we have expanded the exact formula up to $O(r)$ and ignored small sub-leading terms of $O(r^2)$. Again we see that for Type-II-HEa Schemes with IH, the CP-asymmetry $\overline{\epsilon}_1$ only contains the Majorana phase ϕ; it is independent of s_x and the Dirac phase δ. A new feature is the nontrivial dependence on $\rho = \sqrt{M_2/M_1}$ for $\rho \neq 1$.

● *Type-II Schemes with Normal Hierarchy*

For the NH scenario, we derive the CP-asymmetry of leptogenesis for Type-II-HEa Schemes,

$$
\overline{\epsilon}_1 = \frac{-\widehat{s}_{b'} s_s r^{1/4} + (\rho - \rho^{-1}) s_x \cos\frac{\phi+2\delta}{2}}{\rho(r s_s^2 + s_x^2) + \rho^{-1} s_s^2 \sqrt{r} - \widehat{s}_{b'} 2 r^{1/4} s_s s_x \cos\frac{\phi+2\delta}{2}}
$$
$$
\times \ 2 s_x \sin\frac{\phi + 2\delta}{2} \ + O(\lambda^2) \,.
$$
(36)

A similar feature to Type-Ia with NH [cf. (27)] is that the CP-asymmetry $\overline{\epsilon}_1$ in (36) also solely depends on the phase-combination $\phi + 2\delta$.

We have analyzed the prediction of baryon asymmetry η_B in Type-II-HEa schemes. For the IH scenario, we first plot the asymmetry η_B versus the low energy Majorana phase ϕ in Fig. 2(a). Unlike the Type-I schemes [cf. Eqs. (23)-(24)], we find η_B to be very sensitive to the mass ratio $\rho^2 = M_2/M_1$ and the sign $\widehat{s}_{b'}$, according to Eqs. (33) and (35). We show, in Fig. 2(b), the correlation between the leptogenesis scale M_1 and the $0\nu\beta\beta$ decay observable $|\mathfrak{M}_{ee}|$ where we have varied the mass ratio $M_2/M_1 = 5 - 100$ in addition to scanning all ν-oscillation parameters within 95% ranges. We find that M_1 is always above 4×10^{12} GeV and largely falls below 10^{14} GeV.

For the NH scenario, Fig. 2(c) plots η_B versus $\sin\theta_x$ with different inputs of the ratio M_2/M_1 and sign $\widehat{s}_{b'}$, which can be compared with the Type-Ia in Fig. 1(c). Fig. 2(d) demonstrates the correlation between the mass M_1 and the J parameter by scanning $M_2/M_1 \in (5, 100)$ and all other observables within 95% C.L. ranges. Unlike Fig. 1(d), the distribution of M_1 has no clear peak around
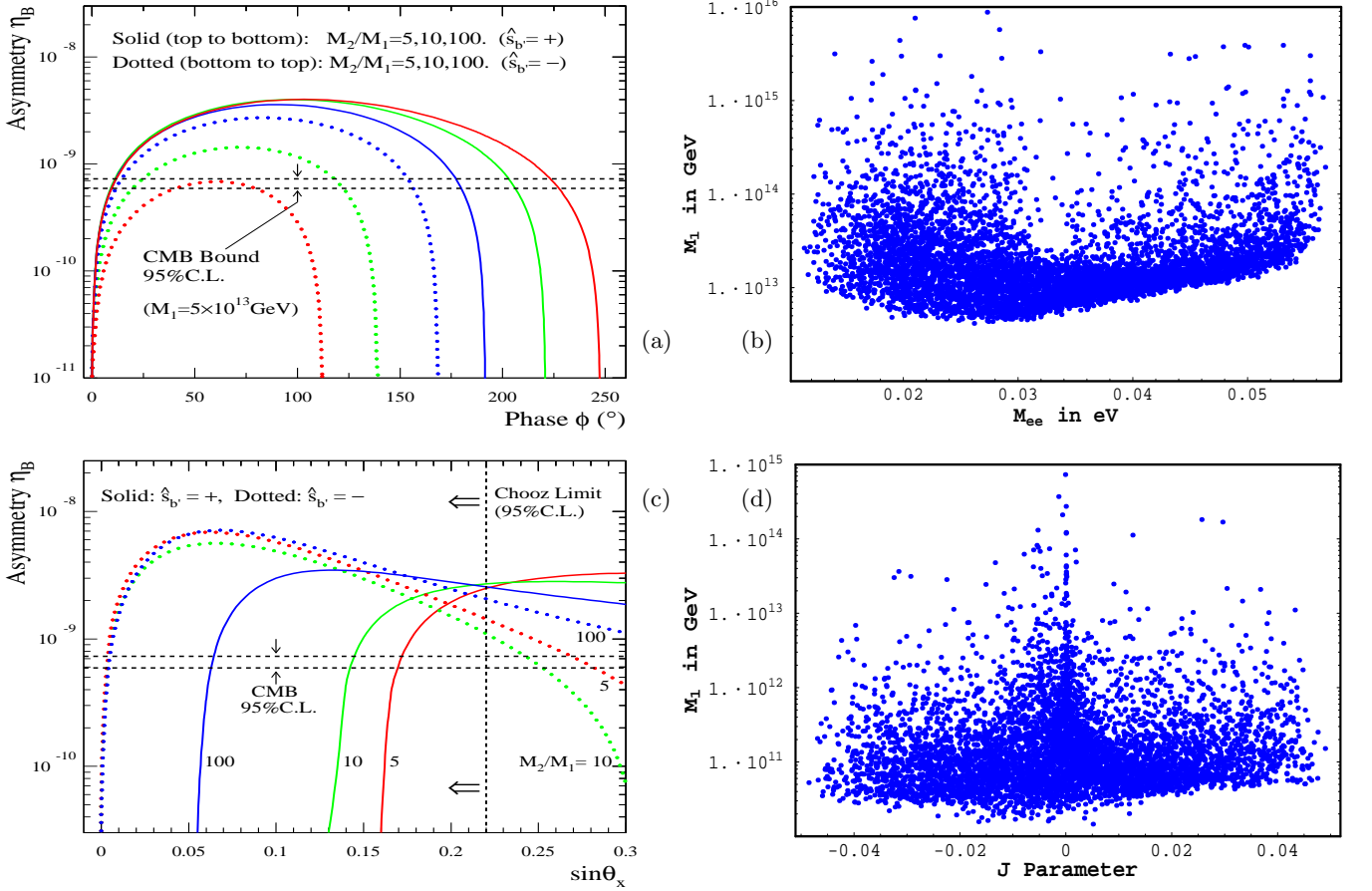
FIG. 2: Analysis for Type-II-HEa. (a) Baryon asymmetry η_B vs. the low energy Majorana phase ϕ in the IH scenario, with best-fit values of ν-oscillation observables as inputs; different curves show the significant effects of varying the ratio M_2/M_1 and the sign $\widehat{s}_{b'}$. (b) Leptogenesis scale M_1 vs. the $0\nu\beta\beta$-decay observable M_{ee} ($\equiv |\mathfrak{M}_{ee}|$) with IH (for $\widehat{s}_{b'} = +$), where we have varied $\phi \in (0, 2\pi)$ and $M_2/M_1 \in (5, 100)$, imposed the 95% C.L. CMB limit and scanned the 95% C.L. ranges of $(\Delta_s, \Delta_a, \theta_s, s_x)$, with 1.2×10^4 samples. (c) Asymmetry η_B vs. $\sin\theta_x$ in the NH scenario, with best-fit values of the ν-oscillation observables as inputs; also $M_1 = 5 \times 10^{11}$ GeV and $\phi + 2\delta = 135°$ are chosen for illustration. (d) Mass scale M_1 vs. CP-violation observable J in the NH scenario (for $\widehat{s}_{b'} = +$), where we have varied $(\delta, \phi) \in (0, 2\pi)$, and scanned the 95% C.L. ranges of the η_B and ν-oscillation parameters, with 1.2×10^4 samples.

$J = 0$.

Finally, we comment on the supersymmetry prediction for η_B. SUSY approximately doubles g_{*1} and the function $F(x)$ so that their ratio remains about the same. The conversion factor $\xi/(\xi - 1) \simeq -0.53$ [cf. (5)] is very close to the SM value -0.55. Another effect is the factor $\sin^2\beta < 1$ which arises from the 2-Higgs-doublet structure of the MSSM but is nearly 1 for $\tan\beta \geqslant 5$. We also note that for light neutrinos with a hierarchical mass-spectrum, possible renormalization group running effects for the neutrino mixing angles and phases are generally small [26], and can be directly incorporated in our reconstruction analysis.

5. Summary and Extension

In this Letter we have presented a general formalism to quantitatively reconstruct leptogenesis for the observed matter-antimatter asymmetry (1) in the mini-

mal neutrino seesaw. We have systematically classified and analyzed such minimal seesaw schemes in which *the required high energy CP-asymmetry for leptogenesis is uniquely linked to the low energy CP-phases in the physical MNSP matrix.* Imposing the cosmological bound (1) and the existing neutrino oscillation data, we have analyzed the constraints and correlations of the leptogenesis scale M_1 versus the low enery neutrino observables $|\mathfrak{M}_{ee}|$ (for $0\nu\beta\beta$ decays) and J-invariant (for long baseline ν-oscillations). The minimal schemes of Type-I (with one-texture-zero) and Type-II (with one-equality) result in distinctive predictions for leptogenesis and its link to the low energy CP-violation observables (cf. Figs. 1-2).

We can readily extend our reconstruction formalism to the general three-heavy-neutrino seesaw where $\mathcal{N} = (N_1, N_2, N_3)^T$ and $M_R = \text{diag}(M_1, M_2, M_3)$ in their mass-eigenbasis. Consequently the Dirac mass matrix m_D becomes a 3×3 matrix,

$$m_D = \begin{pmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{pmatrix} = \begin{pmatrix} \zeta_1\overline{a} & \zeta_2\overline{a}' & \zeta_3\overline{a}'' \\ \zeta_1\overline{b} & \zeta_2\overline{b}' & \zeta_3\overline{b}'' \\ \zeta_1\overline{c} & \zeta_2\overline{c}' & \zeta_3\overline{c}'' \end{pmatrix}, \quad (37)$$

where $\zeta_j \equiv \sqrt{m_0 M_j}$ $(j = 1, 2, 3)$, and m_D contains 3 extra independent CP-phases in $(\overline{a}'', \overline{b}'', \overline{c}'')$. A general classification of minimal schemes for (37) and a complete reconstruction of leptogensis will be given elsewhere. Here we comment on the thermal leptogenesis with a hierarchical mass-spectrum of $\mathcal{N}$ such as, $M_1 \ll M_2 \ll M_3$, which is sometimes called light sequential dominance [27]. With this mass hierarchy, leptogenesis arises from ϵ_1 via the N_1 decays, which is extended from (10) as

$$\epsilon_1 = \frac{m_0 M_1}{8\pi}\left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 [x_{21}F(x_{21})\,\overline{\epsilon}_{12} + x_{31}F(x_{31})\,\overline{\epsilon}_{13}],$$

$$\overline{\epsilon}_{13} \equiv \frac{\Im\mathfrak{m}\left[\left((m_D^\dagger m_D)_{13}\right)^2\right]}{(m_0 M_3)(m_D^\dagger m_D)_{11}} = \frac{\Im\mathfrak{m}\left[(\overline{a}^*\overline{a}'' + \overline{b}^*\overline{b}'' + \overline{c}^*\overline{c}'')^2\right]}{|\overline{a}|^2 + |\overline{b}|^2 + |\overline{c}|^2},$$

$$(38)$$

where the formula for $\overline{\epsilon}_{12}$ takes the same form as $\overline{\epsilon}_1$ in (10), and $(x_{21}, x_{31}) \equiv (M_2/M_1, M_3/M_1)$. The entries in the Dirac mass matrix m_D are generally of $O(v)$, except for the possible texture zeros. Thus the mass hierarchy

$M_1 \ll M_2 \ll M_3$ implies

$$\frac{\overline{\epsilon}_{13}}{\overline{\epsilon}_{12}} = \frac{O(\overline{a}'', \overline{b}'', \overline{c}'')^2}{O(\overline{a}', \overline{b}', \overline{c}')^2} = O\left(\frac{M_2}{M_3}\right) \ll 1. \quad (39)$$

Hence, with the above mass hierarchy, the CP-asymmetry ϵ_1 is dominated by $\overline{\epsilon}_{12}$ (contained in the 2-neutrino seesaw), with the $\overline{\epsilon}_{13}$ term suppressed by the factor $M_2/M_3 \ll 1$. Furthermore, the modifications to the reconstruction solutions of $(\overline{a}, \overline{b}, \overline{c})$ and $(\overline{a}', \overline{b}', \overline{c}')$ relative to a minimal (N_1, N_2)-seesaw are also suppressed by $M_2/M_3 \ll 1$. Thus with $M_1 \ll M_2 \ll M_3$, leptogenesis effectively reduces to that of the minimal seesaw in Sec. 2-4,

$$\epsilon_1 = \frac{m_0 M_2}{8\pi}\left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 F\left(\frac{M_2}{M_1}\right)\overline{\epsilon}_{12}\left[1 + O\left(\frac{M_2}{M_3}\right)\right],$$

$$(40)$$

which allows the direct application of our Minimal Seesaw Schemes (Sec. 2-4) to the 3-neutrino seesaw at the leading order of the M_2/M_3 expansion.

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